NUMBER SYSTEM

Interesting Facts

- Any terminating or recurring decimal is a rational number.
- The product of 'n' consecutive natural numbers is always divisible by n! **For Example**: 11 * 12 * 13 * 14 is completely divisible by 4! Consider 1 * 2 * 3 * 4 * 5 * 6, as per above result 1 * 2 * 3 * 4 * 5 * 6 is divisible by 6! Now take (1 * 2 * 3) * (4 * 5 * 6) , each of this is divisible by 3!
- The product of all odd (even) integers up to an odd (even) positive integer n is called the **double factorial** of n. **For Example:** 9!! = 1 * 3 * 5 * 7 * 9 = 945, 8!! = 2 * 4 * 6 * 8 = 384
- A factorion is a natural number that equals the sum of the factorials of its decimal digits. For example, 145 is a factorion because 1! + 4! + 5! = 1 + 24 + 120 = 145. There are just four factorions (in base 10) and they are 1, 2, 145 and 40585
- 142857 is a CYCLIC NUMBER since its digits are rotated around when multiplied by any number from 1 to 6.
- Consecutive integers can be written as n, n+1, n+2, n+3 ……. and so on.
- Consecutive odd and even integers are written as n, n+2, n+4………..and so on.
- The product of 3 consecutive natural numbers is divisible by 6. If the 1st number is an even number, then the product is divisible by 24 also.
- The product of a n-digit and a k-digit number can be at maximum of (n+k) digits. (For Example, product of 999 * 99 will be of 5 digit and is equal to 98901)
- Rapidly Divide by 9, 99, 999 (Numbers smaller than 9, 99, 999) **4 / 9 = 0.4444444, 62 / 99 = 0.62626262, 5 / 999 = 0.005005005**

How many times a digit 'n' appears in a certain range (except 0)?

- From 1 – 9, the digit 'n' is used 1 time
- From 1 – 99, the digit 'n' is used 20 times.
- From 1 – 999, the digit 'n' is used 300 times
- From 1 – 9999, the digit 'n' is used 4000 times and so on

Number of digits between two numbers:

- Number of single digit natural numbers (i.e. from 1 – 9) = 9
- Number of two digit natural numbers (i.e. from 10 – 99) = 90
- Number of three digit natural numbers (i.e. from 100 – 999) = 900
- Number of 4 digit natural numbers (i.e. from 1000 to 9999) = 9000 and so on…..

Just observe the pattern of series in above two illustrations

1, 20, 300, 4000, 50000, ………….

9, 90, 900, 9000, 90000, ………….

For Example: If you write 1st 252 natural numbers in a straight line, how many times do you write the digit 4?

Solution: In the 1st 99 natural numbers, digit 4 comes 20 times. Similarly, from 100 to 199, digit 4 comes 20 times. Now from 200 to 299, digit 4 comes again 20 times out of which we need to subtract 5 numbers (254, 264, 274, 284 and 294). Therefore, total number of times that we write the digit 4 = 20 + 20 + 20 – 5 = 55.

Numbers Magic – 1

3 x 37 = 111
33 x 3367 = 111,111
333 x 333667 = 111,111,111
3333 x 33336667= 111,111,111,111
33333 x 3333366667= 111,111,111,111,111
And so on.....

Numbers Magic – 2

6 when multiplied with 7 give 4 and 2

6 x 7 =42
66 x 67 = 4422
666 x 667 = 444222
6666 x 6667= 44442222

Numbers Magic – 3

1+2+3+ . . . . + 10 = 55
11 + 12 + ... + 20 = 155
21 + 22 + ... + 30 = 255
31 + 32 + ... + 40 = 355
And so on....

Example 1: Lets' say a CAT admit card has 6-digit number. If we multiply the 1st two digits with 3, we get all 1's. If we multiply the next two digits with 6, we get all 2's. If we multiply the last two digits with 9, we get all 3's. Find the number on the admit card.

Sol: 111/3 = 37; 222/6 = 37; 333/9 = 37 Hence the 6-Digit Number is **373737**

Example 2: Find the 281st term of the sequence - ABBCCCDDDDEEEEE.....

Sol: A appears 1 time; B for 2 times, C for 3 times and so on......Now from above mentioned series we get that **1+2+3+4+..........+20 = 55 + 155 = 210 and 210 + 21 + 22 + 23 = 276**, we need to find 281...so 276 + 24 = 300 which covers the 281st part. Hence, 24th letter will be 281st term of sequence. Hence, the answer is **"X"**

SQUARE NUMBER CHECK

1. Square of any natural number can be written in the form of **3n or 3n+1 or 4n or 4n+1**.
2. Any square number never ends [unit digit] with **2, 3, 7 or 8.**
3. Ten's digit of any perfect square number is always **even** [except when unit digit of that square number is '6']. For example. 9^2 = **8**1, 19^2 = 3**6**1, etc except 14^2 = 1**9**6, 16^2 = 2**5**6, etc
4. The digital sum of any square number will always be **1 or 4 or 7 or 9.**
   a. For example; 14^2 = 196. Digital Sum of 196 = 1 + 9 + 6 = 16 = 1 + 6 = **7**
   b. 88^2 = 7744. Digital Sum of 7744 = 7 + 7 + 4 + 4= 22 = 2 + 2 = **4**

PRIME NUMBERS - Properties

- 1 is not a prime, 2 is the smallest and the only even prime number.
- 3, 5, 7 are the only 3 consecutive odd prime numbers.
- Every prime number greater than 3 can be written in the form of (6k+1) or (6k-1). However, the converse of this is not always true. For e.g., even though 25 can be expressed in the form 6k + 1 where k = 4, it is not a prime number.
  - Prove It!
  - Take 5 = 6 x 1 -1
  - Take 7 = 6 x 1 + 1
  - Take 47 = 6 x 8 – 1
- The remainder of the division of the square of a prime number p ≥ 5 divided by 12 or 24 is 1.
- There are 25 prime numbers in between 1 to 100.
- All prime numbers except 2 and 5 ends in 1, 3, 7 or 9
- Two numbers are co prime if there HCF is 1, for example 21 & 25
- The Squares of prime numbers have exactly three divisors, i.e., 1, the prime number and the square itself.

Number Range	Number of Primes
1-100	25
101-200	21
201-300	16
301-400	16
401-500	17
501-600	14
601-700	16
701-800	14
801-900	15
901-1000	14

To Check whether a number is PRIME or NOT ?

- **Primality Test**

For example, to check whether 179 is prime or not?

- **Step 1:** If n is not a perfect square, find a suitable value of n' which is a perfect square closest to n and greater than n. (Here, 179 is not a perfect square. 196 is greater than 179 and is the closest perfect square, so n' = 196)
- **Step 2:** $\sqrt{n'} = \sqrt{196} = 14$, all prime numbers below 14 are 2,3,5,7,11,13
- **Step 3:** If given number 179 is divisible by any of the above prime numbers, then the given number is not a prime number
- As 179 is not divisible by any of the listed numbers, so 179 is a prime number

- If a number ends with any number except 1,3,7,9 then it will not be prime number (Except 2 & 5)
- Prime numbers of the form $2^n + 1$ and $2^n - 1$ do exist, 7,31, 257 are some examples

SQUARING TECHNIQUES

Squaring a 2-Digit number can be very easy if you know the square of 50. Also try to remember the squares of the numbers from 1-25

$50^2 = 2500$

Now, $51^2 = (50 + 1)^2$ $52^2 = (50 + 2)^2$

51^2 = 25 00
+ 01 $+(01)^2$
51^2 = 26 01

52^2 = 25 00
+ 02 $+(02)^2$
52^2 = 27 04

62^2 = 25 00 (62 = 50 + 12)
+ 12 $+(12)^2$
62^2 = 38 44 (carry over & add 1 to other side)

47^2 = 25 00 (47 = 50 – 03)
- 03 $+ (03)^2$
47^2 = 22 09

39^2 = 25 00 (39 = 50 – 11)
- 11 $+ (11)^2$
39^2 = 15 21 (carry over & add 1 to other side)

Similar kind of pattern is observed when squaring the numbers near 100; **100^2 = 10000**

101^2 = 100 00
+ 02 $+(01)^2$
101^2 = 102 01

106^2 = 100 00

\+ 12 $+(06)^2$

106^2 = 112 36

99^2 = 100 00

\- 02 $+(01)^2$

99^2 = 98 01

93^2 = 100 00

\- 14 $+(07)^2$

93^2 = 86 49

You must have got the pattern whats happening in case of numbers near to 100, from 100 we are subtracting and adding the double of the added/subtracted number, but in case of 50 we subtract and add the same number with 25.

One can apply it in finding the squares near to 1000 also; ***1000^2 = 1000000***

998^2 = 1000 000

\- 04 $+(02)^2$

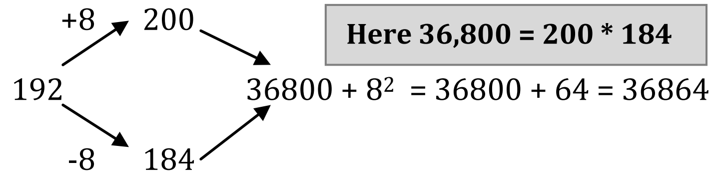
998^2 = 996 004

FINDING 3-Digit Squares

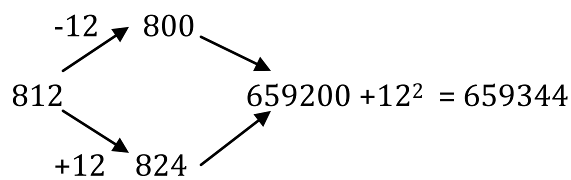
Finding 3 digit squares can be a 2 minute process if you practice the following trick

Suppose you have to find square of 192

Then just round up / down the given number to nearest multiple of 100 as follows



Take another example…….812^2



Try to Find 991^2, 1005^2, 737^2, 563^2

CUBING TECHNIQUES

Suppose you have to find cube of 24

Step 1: Find the ratio of two given numbers i.e. 2 : 4 or 1 : 2

Step 2: write the cube of first digit of the number i.e. 2 * 2 * 2 = 8

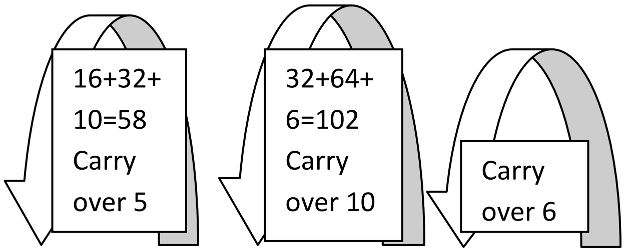
Step 3: write numbers in a row of 4 terms in such a way that the first one is the cube of the first digit and remaining three are obtained in a geometric progression with common ratio as the ratio of the original two digits i.e. the row will be like this

8 16 32 64

Step 4: Write twice the values of 2nd and 3rd terms under the terms respectively in second row

8	16	32	64
	32	64	

Step 5: Add the numbers column wise (Right to Left) and follow carry over process till the end as follows



8	16	32	64
	32	64	
13	**8**	**2**	**4**

So Cube of 24 = 13824

(This method is applicable for finding cubes of 2-Digit Numbers only)

Finding the Cube Root of a Perfect Cube Number

Memorize the first ten cubes, in following manner

Number	Cube	Unit's Digit
1	1	1
2	8	8
3	27	7
4	64	4
5	125	5
6	216	6
7	343	3
8	512	2
9	729	9
10	1000	0

Now, Ask your friend to secretly pick any two-digit number and then have him or her use a calculator to cube it. Let's say he picks 76. So using the calculator he computes 76 * 76 * 76

Then, He asks you to tell him the cube root of 438, 976.

You can give him the answer in 5 seconds:

Number	Last 3 digits	First 3 digits
438, 976	976	438

Now look in the table above and check which cube is less than the first 3 digits of the given number, its 343, and 343 is the cube of 7………so your ten's digit of the cube root is 7

Now look at the unit's place of last 3 digits of the given number, its 6………..check from the above table which number from 1-10 gives you the unit's digit 6 in cube……………it is 6^3 = 216, hence the unit's digit of the cube root is 6

So, you can tell your friend that $\sqrt[3]{438,976} = 76$

DIVISIBILITY RULES

DIVISIBILITY TEST FOR PRIME NUMBERS

The process to check whether a natural number is divisible by a prime number p, is as follows:
Step 1: Find the smallest multiple of p, which is of the form (10k + 1) or (10k – 1), where k is a natural number.
Step 2: If it is of the form (10k – 1), then we check whether (x + ky) is divisible by p or not, and if it is of the form (10k + 1) then we check whether (x – ky) is divisible by p or not. Here y is the unit's digit of the given prime number and x is the number formed by rest of the digits of the given prime number. The process can be repeated if the given number is having more than 3 digits.

Example: Is 4012 Divisible by 17?

Solution: The least multiple of 17 of the form (10k ± 1) is 51, where k = 5, with positive sign.

Thus the divisibility rule of 17 is to check whether (x – 5y) is divisible or not as discussed below

For a number to be divisible by:	It should satisfy the following conditions:
2	The last digit should be divisible by 2; i.e. it should be 2, 4, 6, 8 or 0.
4	The number formed by the last 2 digits should be divisible by 4.
8	The number formed by the last 3 digits should be divisible by 8
16	The number formed by the last 4 digits should be divisible by 16
3	The sum of its digits should be divisible by 3.
9	The sum of its digits should be divisible by 9.
5	The last digit should be either 5 or 0.
10	The last digit should be 0.
6	It should be divisible by both 2 and 3
12	It should be divisible by both 3 and 4.
14	It should be divisible by both 2 and 7.
15	It should be divisible by both 3 and 5.
18	It should be divisible by both 2 and 9.
20	It should be divisible by both 4 and 5
7	$(x - 2y)$ should be divisible by 7.*
13	$(x + 4y)$ should be divisible by 13.*
17	$(x - 5y)$ should be divisible by 17.*
19	$(x + 2y)$ *should be divisible by 19.**
* Here, y is the last digit and x is the number formed by the remaining digits. These 4 results can be derived using the divisibility test for prime numbers. Example: **665 is div by 19?** ***Here y = 5 & x = 66. Then x + 2y = 76 and 76 is div by 19. Hence 665 is div by 19***	
11	The difference of the sum of the digits in odd positions and the sum of the digits in even positions should be divisible by 11.

Divisibility by Numbers of the form $10^n - 1$ / $10^n + 1$

Divisibility by 9 Let's Say X=abcdefgh is divisible by 9 or not? Now, $9 = 10^1 - 1$ So, Sum of digits of X is done 1 at a time, that is, a + b + c + d + e + f + g + h = P If P is divisible by 9, then X is also divisible by 9	**Divisibility by 99** Let's Say X=abcdefgh is divisible by 99 or not? Now, $99 = 10^2 - 1$ So, Sum of digits of X is done 2 at a time(starting from last), that is, ab + cd + ef + gh = P If P is divisible by 99, then X is also divisible by 99	**Divisibility by 999** Let's Say X=abcdefgh is divisible by 999 or not? Now, $999 = 10^3 - 1$ So, Sum of digits of X is done 3 at a time(starting from last), that is, ab + cde + fgh = P If P is divisible by 999, then X is also divisible by 999
Divisibility by 11 Let's Say X=abcdefgh is divisible by 11 or not? Now, $11 = 10^1 + 1$ So, Alternating Subtraction & Summation of digits of X is done 1 at a time, that is, a - b + c - d + e - f + g - h = P If P is divisible by 11, then X is also divisible by 11	**Divisibility by 101** Let's Say X=abcdefgh is divisible by 101 or not? Now, $101 = 10^2 + 1$ So, Alternating Subtraction & Summation of digits of X is done 2 at a time, that is, ab - cd + ef - gh = P If P is divisible by 101, then X is also divisible by 101	**Divisibility by 1001** Let's Say X=abcdefgh is divisible by 1001 or not? Now, $1001 = 10^3 + 1$ So, Alternating Subtraction & Summation of digits of X is done 3 at a time, that is, ab - cde + fgh= P If P is divisible by 1001, then X is also divisible by 1001

- *X is divisible by all factors of 9, 99 & 999. Hence, the same divisibility test also work for 3, 11, 27, 37, 41, 271 and others*
- *X is divisible by all factors of 11, 101 & 1001. Hence, the same divisibility test also work for 7,11, 13, 73, 137 and others too*

Apti-Trick 1: Dividing Numbers by 5, 25, 125

*32 / 5----Multiply 32 by 2 = 64-----Put 1 decimal point from the end 64, i.e., **6.4**----- Hence 32 / 5 = 6.4.*
*Similarly, 133/5 -----133 * 2 =266-----1 decimal point=**26.6**, hence the answer*

*44 /25-------Multiply 44 by 4 = 176----put decimal point after 2 places from the end, i.e., **1.76***

*90 /125------Multiply 90 by 8 = 720--put decimal point after 3 places from the end, i.e., **0.720***

Apti-Trick 2: Dividing Numbers by 7 is very easy

*1 / 7 = **0.142857***
*2 / 7 = **0.285714***
*3 / 7 = **0.428571***
*4 / 7 = **0.571428***
*5 / 7 = **0.714285***
*6 / 7 = **0.857142***
*7 / 7 = **1.000000***
*8 / 7 = **1.142857 = 1 + 1/7***
*9 / 7 = **1.285714 = 1 + 2/7***
*10 / 7 = **1.428571 = 1 + 3/7***
'
'
and the list goes on repeating

So, what is happening here
Just remember 1/7 = 0.142857
Then, for 2/7, find the next smallest digit after 1 in 0.142857, it is 2
write the sequence of numbers after 2
0.285714, Hence it becomes value of 2/7
and the list go on like this

Factors / Divisors of a Given Composite Number

Suppose That a Composite Number ‘n’ is given
Factorize the number n into its prime factors, i.e. $n = a^x b^y c^z$ **...** where a, b, c are prime numbers

Number of Divisors	**(x + 1)(y + 1)(z + 1)……** For Eg : $12 = 2^2 * 3^1$ Number of divisors = (2 + 1)*(1 + 1) = 3 x 2 = 6
Number of Even & Odd Divisors	Number of even divisors = **(x)(y + 1)(z + 1)……** Number of odd divisors = **(y + 1)(z + 1)……** For Eg : $12 = 2^2 * 3^1$ Number of even divisors = (2)*(1 + 1) = 2 * 2 = 4
Sum of Divisors	$\frac{(a^{x+1}-1)(b^{y+1}-1)(c^{z+1}-1)....}{(a-1)(b-1)(c-1)....}$
Product of Divisors	$= [n]^{\frac{(a+1)(b+1)(c+1)}{2}}$ $= [a^x b^y c^z]^{\frac{(a+1)(b+1)(c+1)}{2}}$
Number of ways of expressing a given number as a product of two factors	**½ (x + 1)(y + 1)(z + 1)…** (If n is not perfect square) **½ [(x + 1)(y + 1)(z + 1)… + 1]** (if n is perfect square)
Number of ways of expressing a given number as a product of two Co-Primes	$[2]^{m-1}$ For Eg : $120= 2^3 * 3 * 5$ It has 3 distinct prime factors, 2, 3 & 5 (i.e. *m* = 3).So, No of ways 120 can be written as product of two co-primes = $[2]^{3-1}$ = 4
Number of sets of factors which are co-prime to each other	**(x+1)(y+1)(z+1)…..-1+ (xy + yz + zx + …..)** For Eg : $12 = 2^2 * 3^1$ Number Sets of Co-prime factors = (2+1)*(1+1) - 1 + (2 * 1) = 5 + 2 = 7

HCF/LCM of Numbers

Most of us are aware about the Long Division Methods and Prime Factorization Methods for finding the HCF and LCM of numbers. Here are some methods other than those which are relatively quick and easy.

Example 1:

Find the HCF and LCM of 42 and 96.
Sol.
We make a fraction and simplify it as much as we can as follows:

$$\frac{42}{96} = \frac{21}{48} = \frac{7}{16}$$

(Be sure that no more simplification is possible)

Therefore, HCF = 42 / 7 = 6
LCM = 42 X 16 = 672.
It is advisable to double check
HCF = 6 = 96 / 16...........YES
LCM = 672 = 96 X 7............YES

Example 2:

Find the HCF and LCM of 120 and 36.

$$\frac{120}{36} = \frac{60}{18} = \frac{30}{9} = \frac{10}{3}$$

Therefore, HCF = 120 / 10 = 12
LCM = 120 X 3 = 360
Double check ! ! !
HCF =12 = 36 / 3...........YES
LCM = 360 = 36 X 10............YES

Another Interesting Way to Find HCF of more than two given numbers

Suppose you have been asked to find HCF of 220, 280, and 375.

Sol.

220 280 375

Find the difference between any two numbers among the given three, i.e.

Let's say 280 – 220 = 60

Then, write the factors/divisors of 60 in descending order as follows:

60 = 60, 30, 20, 15, 12, 10, 6, 5, 4, 3, 2, 1

Then check which divisor exactly divides all the given three numbers (220, 280, and 375) in the same descending order. The largest divisor which will first exactly divide all the given three numbers will be the HCF of the numbers.

In this case, the HCF will be 5, as 5 is the first largest divisor which will exactly divide all three numbers.

(This Technique is very much handy in cases where the difference between two numbers will be prime number. In such cases, you will have to check the divisibility with only the prime number so found, if it is exactly dividing the given numbers, then that prime number will be the HCF, otherwise the HCF of given Numbers will be 1)

Example:
Find HCF of 483, 502 and 546?
Sol.
Here, 502 – 483= 19 (Prime Number)
19 don't divide all three given numbers.
Hence, HCF = 1

APPLICATIONS of HCF / LCM of NUMBERS	
Find the GREATEST NUMBER that will *exactly* divide x, y, z.	Required number = H.C.F. of x, y, and z
Find the GREATEST NUMBER that will divide x, y and z leaving remainders a, b and c respectively.	Required number = H.C.F. of (x – a), (y – b) and (z – c).
Find LEAST NUMBER which is *exactly* divisible by x, y and z.	Required number = L.C.M. of x, y and z
Find LEAST NUMBER which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that (x – a) = (y – b) = (z – c) = K (say) ∴ Required number = (L.C.M. of x, y, z) – K
Find LEAST NUMBER which when divided by x, y and z leaves the same remainder 'r' in each case.	Required number = (L.C.M. of x, y, z) + r
Find the GREATEST NUMBER that will divide x, y and z leaving the same remainder in each case.	Required number = H.C.F of (x – y), (y – z) and (z – x).

Properties of HCF / LCM

1. Product of two numbers = L.C.M. × H.C.F.
2. **If ratio of numbers is a : b and H is the HCF of the numbers Then**
   LCM of the numbers = H × a × b
   = HCF × Product of the ratios.
3. H.C.F of fractions = $\left(\frac{\textbf{HCF of Numerators}}{\textbf{LCM of Denominators}}\right)$
4. L.C.M. of fractions = $\left(\frac{\textbf{LCM of Numerators}}{\textbf{HCF of Denominators}}\right)$
5. If HCF (a, b) = H1 and HCF (c, d) = H2, then HCF (a, b, c, d) = HCF (H1, H2).
6. LCM is always a multiple of HCF of the numbers

Examples of Problems based on HCF/LCM

Ex.1) In a farewell party, some students are giving pose for photograph, If the students stand at 4 students per row, 2 students will be left if they stand 5 per row, 3 will be left and if they stand 6 per row 4 will be left. If the total number of students are greater than 100 and less than 150, how many students are there?

Sol)
If 'N' is the number of students, it is clear from the question that if N is divided by 4, 5, and 6, it produces a remainders of 2, 3, & 4 respectively. Since (4 – 2) = (5 – 3) = (6 – 4) = 2, the least possible value of N is LCM (4, 5, 6) – 2 = 60 – 2, = 58. But, 100 < N < 150. So, the next possible value is 58 + 60 = **118.**

Ex.2) There are some students in the class. Mr X brought 130 chocolates and distributed to the students equally, then he was left with some chocolates. Mr Y brought 170 chocolates and distributed equally to the students. He was also left with the same no of chocolates as Mr X was left. Mr Z brought 250 chocolates, did the same thing and left with the same no of chocolates. What is the max possible no of students that were in the class?

Sol)
The question can be stated as, what is the highest number, which divides 130, 170 and 250 gives the same remainder, i.e.
HCF [(170 −130), (250 −170),(250 −130)] .i.e.
HCF (40, 80, 120) = 40.
Hence, at maximum 40 students were there in the class

APPROXIMATING SQUARE ROOTS

Unknown Square root

$$= \text{Known Sq Root} - \frac{\text{Known Sq} - \text{Unknown Sq}}{2 * \text{Known Sq Root}}$$

(The known square selected should be as near to the given unknown number as possible. More closely the selected known square number is, more correct the approximation is)

For Example: Find $\sqrt{58}$

Solution:

64 is the closest perfect square known near to 58

Just put the values in the above mentioned formula

$$\sqrt{58} = \sqrt{64} - \frac{64 - 58}{2 * \sqrt{64}}$$

$$\sqrt{58} = 8 - \frac{6}{2 * 8}$$

$$\sqrt{58} = 8 - \frac{3}{8}$$

$$\sqrt{58} = 7.625$$

Finding in Calculator we get $\sqrt{58}$ = 7.615

So, our approximation was very much near to the answer

You can try it for 2-digit, 3-digit, 4 digit……. roots also

Digital Sum of a Number

If we add up the digits of a number until there is only one number left, we have found out what is called the **Digital Sum**. Digital Sum of a number is also known as the **Digital Root** of the number

Example1: What is the digital sum of 78562? **Solution:**

Digital Sum of 78562 = 7+8+5+6+2 = 28 = 2+8 = 10 = 1+0 = 1

Also the digital sum of the product of two numbers is equal to the product of the digital sums of the two numbers. Let's check this with an example-
The product of 795 and 36 is 28620.
Digital sum of 795 = 7+9+5 = 21 = 2+1 = 3
Digital sum of 36 = 3+6 = 9
Product of these digital sums = 9x3 = 27 = 2+7 = 9
Digital sum of 28620 = 2+8+6+2+0 = 18 = 1+8 = 9

Applications of Digital Sum: This concept can be used almost everywhere! This can be helpful in the rapid checking of calculations while multiplying numbers. It greatly simplifies the calculation of Simple and Compound interest which we will discuss in a later unit. We can use this to check whether a number is a perfect square or not as the digital sum of perfect squares will always be 1, 4, 7 or 9.

Example 2: (986*137) + (986*863) = _________

a)985000 b) 986000 c) 1000000 d)999000

Sol) Calculating the digital sum of each number, we will get an equation in the following terms:

(5*2) + (5*8) = (10) + (40) = 1 + 4 = 5

Now check the options, option b gives you the digital sum of 5. **Hence the answer is option (b)**

CONCEPT of CYCLICITY / Power Cycle

Cyclicity of any number is about the last digit of the number and how they appear in a certain defined manner.

For Example:
What is the last digit / unit digit of 27^{1001}?

These type of questions can be solved by knowing the cyclicity of numbers from 1-10.

Table Showing Unit Digit of a Number for different exponents:

Unit Digit (N)	N^1	N^2	N^3	N^4	N^5	Cyclicity
1	1	1	1	1	1	1
2	2	4	8	6	2	4
3	3	9	7	1	3	4
4	4	6	4	6	4	2
5	5	5	5	5	5	1
6	6	6	6	6	6	1
7	7	9	3	1	7	4
8	8	4	2	6	8	4
9	9	1	9	1	9	2
10	0	0	0	0	0	1

So, here we see that for every number from 1-10, unit digit for their different exponents repeats after certain intervals and this interval is called **cyclicity.** It is very helpful in finding the units digit of any number.

TEN's DIGIT CYCLICITY

Just as the unit's digit of a number that is raised to varying powers repeats itself in cycles, similarly the ten's digit also shows cyclicity.

Digits	Tens Digit Cyclicity
2, 3, 8	20
4, 9	10
6	5
7	4
5	1

REMAINDER THEOREM

Ground Rules

*Remainder [(a * b) / c]*
*= Remainder [a / c] * Remainder [b / c]*
Remainder [(a + b) / c]
= Remainder [a / c] + Remainder [b / c]
Remainder [(a - b) / c]
= Remainder [a / c] - Remainder [b / c]

For Example:
What is the remainder when 1998 x 1999 x 2000 is divided by 7?
Solution:
1998/7 gives Remainder 3
1999/7 gives Remainder 4
2000/7 gives Remainder 5

So, Final Remainder = (3*4*5) / 7 = 60 /7 = 4

Example:
What is the remainder when 32^{32} is divided by 9?
Solution: Using the principles of cyclicity
$32^{32} = (2^5)^{*32} = (2^{160})$
Now 2^{6k} gives remainder 1 when divided by 9
$(2^{160}) = 2^{156} \times 2^4 = 2^{(6*26)} \times 2^4$
$2^{(6*26)}$ gives remainder 1 and 2^4 gives remainder 7 when divided by 9 respectively
Hence final remainder = 7

Try for These 3 cases

$32^{32^{32}}$ is divided by 7 ?

$25^{25^{25}}$ is divided by 9 ?

$97^{97^{97}}$ is divided by 11 ?

Case of Negative Remainders

Suppose you have to find remainder when 7^{52} is divided by 2402?

7^4 = 2401 will give you remainder (-1) when divided by 2402. So, we can say that the twice of this power will give you remainder (+1), i.e. 7^8 when divided by 2402 will give you remainder (+1).

$7^{52} = 7^{48} * 7^4 .....7^{48} = 7^{(8*6)}$ will give remainder (+1) & 7^4 will be left with remainder (-1) when divided by 2402. Hence, Final Remainder = 2402-1= **2401** (because remainder is -1, we will subtract it from 2402 to get the final remainder)

Dividend is of the form $a^n + b^n$ / $a^n - b^n$

Rule 1: $a^n + b^n$ is divisible by a + b when n is **ODD**
Rule 2: $a^n - b^n$ is divisible by a + b when n is **EVEN**
Rule 3: $a^n - b^n$ is **ALWAYS** divisible by a - b

For Example

$5555^{2222} + 2222^{5555}$ divided by 7 gives remainder?

Solution: 5555 & 2222 gives remainder 4 and 3 respectively when divided by7.

$\mathbf{5555^{2222} + 2222^{5555}}$ divided by7
$\mathbf{= 4^{2222} + 3^{5555}}$ divided by7
$\mathbf{= (4^2)^{*1111} + (3^5)^{*1111}}$ divided by7
Now it is of the form $a^n + b^n$ when n is **ODD.** So, the expression will be divisible by $\mathbf{4^2 + 3^5 = 16 + 243 = 259}$, which when divided by 7 gives remainder 0.
Hence, Final remainder = 0

FERMAT's REMAINDER THEOREM

Let *p* be a prime number and *a* be a number that is co-prime to *p.* Then, the remainder obtained when $\mathbf{a^{p-1}}$ is divided by **p** is 1.

For Example:
Find the remainder when $2^{100} + 2$ is divided by 101?
Solution:
101 is a prime number and 2 is co-prime to 101.
$2^{100} = 2^{101-1}$ = This when divided by 101 gives remainder 1 according to Fermat's Theorem.
Also, 2 when divided by 101 give 2 as remainder.
Hence, Final Remainder = 1 + 2 = 3

WILSON's REMAINDER THEOREM

If p is a prime number, then (p-1)! + 1 is divisible by p.
For Example:
Find the remainder when 16! Is divided by 17?
Solution:
17 is a prime number.
16! = (16! + 1) – 1 = [(17-1!) +1] – 17+16
According to Wilson's Theorem [(17-1!) +1] is divisible by 17.
Hence, Final remainder = 16

- $\frac{(a+1)^n}{a}$ will always give a remainder of **1** for all natural numbers **a and n**
- $\frac{(a)^n}{a+1}$ will always give a remainder of **1** when n is **EVEN** and reminder of **a** when n is **ODD**
- The remainder when f(x) = a + bx + cx^2 + dx^3 +.. is divided by x – a is f(a)
- If a number *x* is divided by *n* which leaves a remainder *y*, then this can be represented in terms of mod operator as: ***x* mod *n* = *y***

Trick for Finding the Last / Unit Digit

$\{2, 4, 6, 8\}^{4k}$ gives Unit Digit **6**

$\{1, 3, 5, 7, 9\}^{4k}$ gives Unit Digit **1**

Example1: Find the unit digit for:-

1932^{139}

1943^{147}

1538^{113}

Sol) $1932^{139} = 2^{139}$

Now, 139 can be written as 136 + 3 & 136 is divisible by 4 (Distribute the given power in terms of 4k always)

$2^{139} = 2^{136} * 2^3 = 6 * 8 = 48$ = Unit Digit = 8 (2^{136} is equivalent to 2^{4*34} = this will give unit digit 6 and 2^3 will give unit digit 8)

Also, **$1943^{147} = 3^{147}$** $= 3^{144} * 3^3 = 1*7$ = Unit digit 7

$1538^{113} = 8^{113}$ $= 8^{112} * 8^1 = 6*8$ = Unit Digit 8

Solve for Unit Digit of the Following

$2372^{113} + 133^{147} + 177^{143} + 2322^{199}$

Finding Last Two Digits Shortcut Techniques

Let's classify this topic into Finding-

- Last two digits of numbers which end in 1
- Last two digits of numbers which end in 3, 7 and 9
- Last two digits of numbers which end in 2
- Last two digits of numbers which end in 4, 6 and 8

Last two digits of numbers which end in 1

Let's start with an example.
What are the last two digits of 31^{67}?
Solution: The Unit digit is equal to 1. Multiply the tens digit of the number (3 here) with the last digit of the exponent (7 here) to get the tens digit. Last two digits are 11.

Example 2:

What are the last two digits of 141^{22}?
Solution: The last two digits depend only on the last two digits (41 here). The Unit digit is equal to 1. Multiply the tens digit of the number (4 here) with the last digit of the exponent (2 here) to get the tens digit. Last two digits are 81.

Last two digits of numbers which end in 3, 7 & 9

Convert the number till the number gives 1 as the last digit and then find the last two digits according to the previous method.

Power Cycle of 3 – 3, 9, 7, 1

Power Cycle of 7 – 7, 9, 3, 1

Power cycle of 9 – 9, 1, 9, 1

So we see that 1 comes as the last digit in the power cycles of all these three numbers. Therefore if we convert these numbers into the format of 3^{4k}, 7^{4k}, 9^{2k} then the last digit becomes 1 and we can apply our method.

Let's understand this with an example:
Find the last two digits of 33^{288}.
Sol) $33^{288} = (33^4)^{72}$

Last two digits of $(33^4) = 33^2 * 33^2 = 1089 * 1089$
Last two digits of (89 x 89) = 21
so, $33^{288} = (33^4)^{72} = (21)^{72}$
The last two digits of $(21)^{72} = 41$

Last two digits of 17^2 that is 89 will be same for 33^2,67^2, 83^2,117^2 …….. (33^2, 67^2 are 50±17 and 83^2,117^2 are 100±17) .

Last two digits of 11^2, 39^2, 61^2, 89^2, 111^2 will be the same (39 & 61 are 50±11; 89 & 111 are 100±11)

Last two digits of numbers which end in 2

Last two digits of $(2^{10})^{even}$ is always 76.
Last two digits of $(2^{10})^{odd}$ is always 24.
We just need to remember the above two facts and values of 2^1,2^2 upto 2^{10}. That's it!

We can apply this for any number in the form 2^n.
Find the last two digits of 2^{543}.
Sol)
$2^{543} = (2^{10})^{54} \times 2^3$
Last two digits of $(2^{10})^{54}$ = 76
Last two digits of 2^3 = 08
Last two digits of 2^{543} = 76 x 8 = 08

Last two digits of numbers which end in 4, 6 & 8

Now those numbers which are not in the form of 2^n can be broken down into the form 2^n * odd number. We can find the last two digits of both the parts separately.

Find the last two digits of 56^{283}.
Sol)
$56^{283} = (2^3 \times 7)^{283} = 2^{849} \times 7^{283}$
$= (2^{10})^{84} \times 2^9 \times (7^4)^{70} \times 7^3$
Last two digits = 76 x 12 x 01 x 43 = 16

Number of Exponents/Highest Power/Number of Zeroes

Remembering the following factorials can be useful-

0! = 1; 1! = 1; 2! = 2; 3! = 6; 4! = 24; 5! = 120; 6! = 720; 7! = 5040

The highest power of a prime number p in n! ***is given by the sum of quotients obtained by successive division of n by p.***

Example 1: What is the highest power of 7 that can exactly divide 780!

Sol) [780/7] = 111; [111/7] = 15; [15/7 = 2;

The highest power of 7 = 111 + 15 + 2 = 128

Example 2: Find the exponent of 25 in 250!

Sol) Here 25 is not prime thus we cannot use the above formula directly.

However 25 = 5^2; so we will find the exponent (highest power) of 5 in 250! and then divide the exponent value by the power of 5 which is 2 here to get the exponent of 25.

[250/5] = 50; [50/5] = 10; [10/5] = 2;

The exponent of 5 is 62.
Thus exponent of 25 is 62/2 = 31

Example 3: What is the highest power of 6 that divides 134!

Sol) As 6 is not a prime number, we will divide it into its prime factors. 3 is the bigger prime, so its power will be the limiting factor. Hence, we need to find out the power of 3 in 134!

[134/3] = 44; [44/3] = 14; [14/3] = 4; [4/3] = 1;

Thus the highest power of 3 in 134! = 44 + 14 + 4 + 1 = 63

Example 4: How many zeroes are present in (at the end of) 43!

Sol) Number of ending zeroes is the highest power of 10 that divides n!. 10 is not a prime number and its prime factors are 2 and 5. 5 being the bigger prime, it's power becomes the limiting factor.

[43/5] = 8; [9/5] = 1;
Thus, 43! contains 9 multiples of 5; hence it will have 9 zeroes trailing it.